## Class X

## HALF YEARLY EXAMINATION <br> MATHEMATICS

Time : 3 hrs.
Mark : 80

## General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section $A$ has 20 MCQs carrying 1 mark each
3. Section $B$ has 5 questions carrying 02 marks each.
4. Section $C$ has 6 questions carrying 03 marks each.
5. Section $D$ has 4 questions carrying 05 marks each.
6. Section $E$ has 3 case based integrated units of assessment ( 04 marks each) with sub- parts of the values of 1,1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section $E$
8. Draw neat figures wherever required. Take $\pi=22 / 7$ wherever required if not stated.

## SECTION - A (OBJECTIVE TYPE)

$20 \times 1=20$

1. If $x=2^{3} \times 3^{1} \times 5^{2}, y=2^{2} \times 3^{3}$, then $\operatorname{HCF}\{x, y\}$ is?
a) 12
b) 108
C) 6
d) 36
2. The LCM of two numbers is 90 times their HCF. The sum of LCM and HCF is 1456. If one of the number is 160 , then what is the other number?
a) 120
b) 136
C) 144
d) 184
3. A fraction becomes $4 / 5$ when 1 is added to each of the numerator and denominator. However, if we subtract 5 from each then it becomes $1 / 2$. The fraction is $\qquad$
a) $5 / 8$
b) $5 / 6$
C) $7 / 9$
d) $13 / 16$
4. If the system of equations $2 x+3 y=7$
$2 a x+(a+b) y=28$ has infinitely many solutions, then
a) $a=2 b$
b) $b=2 a$
c) $a+2 b=0$
d) $2 \mathrm{a}+\mathrm{b}=0$
5. If the list price of a toy is reduced by $₹ 2$, a person can buy 2 toys more for $₹ 1360$. The original price of the toy is
a) $₹ 18$
b) ₹ 20
c) $₹ 19$
d) ₹ 21
6. Value(s) of ' $k$ ' for which the quadratic equation $2 x^{2}-k x+k=0$ has equal roots is
a) 0
b) 4
C) 8
d) 0 and 8
7. Common difference of an AP in which $a_{18}-a_{14}=32$ is $\qquad$
a) 8
b) -8
C) -4
d) 4
8. Which term of an AP : 21, 42, 63, 84, $\qquad$ is $210 ?$
a) 9th
b) 10 th
C) 11 th
d) 12 th
9. The point which divides the line segment joining the points $(7,-6)$ and $(3,4)$ in ratio $1: 2$ internally lies in the
a) I quadrant
b) II quadrant
c) III quadrant
d) IV quadrant
10. Distance between the pairs of points $(a, b),(-a,-b)$ is
a) $2 \sqrt{a^{2}+b^{2}}$
b) $\sqrt{a^{2}+b^{2}}$
c) $\sqrt{2 a^{2}+2 b^{2}}$
d) 0
11. Observe the two triangles shown below. Which statement is correct?

a) Triangles are similar by SAS
b) Triangles are similar by SSA
c) Triangles are not similar as sides in proportion
d) No valid conclusion about similar triangles can be made as angle measures are not known
12. If $\triangle \mathrm{ABC} \sim \triangle \mathrm{DFE}, \angle \mathrm{A}=30^{\circ}, \angle \mathrm{C}=50^{\circ} \mathrm{AB}=5 \mathrm{~cm}, \mathrm{AC}=8 \mathrm{~cm}$ and $\mathrm{DF}=7.5 \mathrm{~cm}$. Then which of the following true?
a) $\mathrm{DE}=12 \mathrm{~cm}, \angle \mathrm{~F}=50^{\circ}$
b) $\quad \mathrm{DE}=12 \mathrm{~cm}, \angle \mathrm{~F}=100^{\circ}$
c) $E F=12 \mathrm{~cm}, \angle \mathrm{D}=100^{\circ}$
d) $E F=12 \mathrm{~cm}, \angle \mathrm{D}=30^{\circ}$
13. In the given figure, $A O B$ is a diameter of the circle with centre $O$ and $A C$ is a tangent to the circle at A . If $\angle \mathrm{BOC}=130^{\circ}$, then find $\angle \mathrm{ACO}$.
a) $80^{\circ}$
b) $30^{\circ}$
c) $40^{\circ}$
d) $60^{\circ}$

14. In figure $P Q$ is a tangent at a point $C$ to a circle with centre $O$. If $A B$ is a diameter and $\angle \mathrm{CAB}=30^{\circ}$ find $\angle \mathrm{PCA}$.
a) $60^{\circ}$
b) $45^{\circ}$
c) $15^{\circ}$
d) $70^{\circ}$

15. Two tangents making an angle of $60^{\circ}$ between them are drawn to a circle of radius $\sqrt{3} \mathrm{~cm}$, then find the length of each tangent.
a) 7 cm
b) 10 cm
c) 5 cm
d) 3 cm

16. The value of $\left(\operatorname{Sin} 30^{\circ}+\operatorname{Cos} 30^{\circ}\right)-\left(\operatorname{Sin} 60^{\circ}+\operatorname{Cos} 60^{\circ}\right)$ is
a) -1
b) 0
c) 1
d) 2
17. If $\sin \alpha=1 / 2$, then the value of $(\tan \alpha+\cos \alpha)^{2}$ is
a) $16 / 3$
b) $8 / 3$
c) $4 / 3$
d) $10 / 3$
18. If $\tan (A+B)=\sqrt{3}$ and $\tan (A-B)=\frac{1}{\sqrt{3}}$ find $A$ and $B$.
a) $15^{\circ}, 30^{\circ}$
b) $45^{\circ}, 15^{0}$
c) $30^{\circ}, 45^{\circ}$
d) $30^{\circ}, 30^{\circ}$

In the following questions (No. 19-20) a statement of Assertion followed by a statement of Reason is given. Choose the correct answer out of the following choices. $2 \times 1=2$
a) Both Assertion (A) and Reason (R) are true and Reason is the correct explanation of Assertion.
b) Both Assertion (A) and Reason (R) are true and but Reason is not the correct explanation of Assertion.
c) Assertion (A) is true but Reason (R) is false
d) Assertion (A) is false but Reason (R) is true.
19. Assertion (A) : $x^{2}+7 x+12$ has no real zeroes.

Reason (R) : A quadratic polynomial can have at the most two zeroes.
20. Assertion (A) : $D$ and $E$ are points on the sides $A B$ and $A C$ of a $\triangle A B C$ such that $A B=$ $10.8 \mathrm{~cm}, A D=6.3 \mathrm{~cm}, A C=9.6 \mathrm{~cm}$ and $E C=4 \mathrm{~cm}$ then $D E$ is parallel to BC .

Reason (R) : If a line is drawn parallel to one side of a triangle then it divides the other two sides in the same ratio.

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\text { SECTION - B } \quad 5 \times 2=10
$$

21. Find the LCM and HCF of 120 and 144 by fundamental theorem of arithmetic.
22. In an A.P., if $S_{5}+S_{7}=167$ and $S_{10}=235$, then find the A.P.

## OR

Find the values of $p$ for which the quadratic equation $4 x^{2}+p x+3=0$ has equal roots.
23. If $A(4,3), B(-1, y)$ and $C(3,4)$ are the vertices of a right triangle $A B C$, right angled at $A$, then find the value of $y$.
24. Prove that tangents drawn at the ends of a chord of a circle make equal angles with the chord.


## OR

In the given figures, find the measure of $\angle \mathrm{X}$.

25. Find the value of $\theta$, if, $\frac{\cos \theta}{1-\sin \theta}+\frac{\cos \theta}{1+\sin \theta}=4 ; \theta \leq 90^{\circ}$

## SECTION - C

26. The numbers 525 and 3000 are both divisible by $3,5,15,25$ and 75 . What is the HCF of 525 and 3000 ? Justify your answer.
27. The $p^{\text {th }}, q^{\text {th }}$ and $r^{\text {th }}$ terms of an A.P. are $a, b$ and $c$ respectively, show that $a(q-r)+b(r-p)$ $+c(p-q)=0$.

## OR

If $S_{n}$ denotes, the sum of the first $n$ terms of an A.P. prove that $S_{12}=3\left(S_{8}-S_{4}\right)$.
28. Find the value of $p$ for which the points $(p+1,2 p-2),(p-1, p)$ and $(p-3,2 p-6)$ are collinear.
29. In the given figure, $D E \| A B$ and $F E \| D B$.

Prove that $D C^{2}=C F \times A C$.

30. In $\triangle A B C, A B=A C$. If the interior circle of $\triangle A B C$ touches the sides $A B, B C$ and $C A$ at D, E, F respectively. Prove that E bisects BC.
31. Prove that: $\frac{\sin \theta-\cos \theta}{\sin \theta+\cos \theta}+\frac{\sin \theta+\cos \theta}{\sin \theta-\cos \theta}=\frac{2}{2 \sin ^{2} \theta-1}$

## OR

If in a triangle $A B C$ right angled at $B, A B=6$ units and $B C=8$ units, then find the value of $\sin A . \cos C+\cos A . \sin C$.

## SECTION - D

$4 \times 5=20$
32. Given that $\tan (A+B)=\frac{\tan A+\tan B}{1-\tan A \tan B}$, find the values of $\tan 75^{\circ}$ and $\tan 90^{\circ}$ by taking suitable values of $A$ and $B$.
33. In the given figure, O is the centre of the circle. Determine $\angle \mathrm{APC}$, if DA and DC are tangents and $\angle \mathrm{ADC}=50^{\circ}$.


## OR

The tangent at a point $C$ of a circle and a diameter $A B$ when extended intersect at $P$. If $\angle P C A=110^{\circ}$, find $\angle C B A$.
34. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of $6 \mathrm{~km} / \mathrm{h}$ more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed?

## OR

The total cost of a certain length of cloth is ₹ 200 . If the piece was 5 m longer and each metre of cloth costs ₹ 2 less, the cost of the piece would have remained unchanged. How longer is the piece and what is its original rate per metre?
35. Solve the following pair of equations: $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$ and $\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

## SECTION - E

36. A barrels manufacturer can produce up to 300 barrels per day. The profit made from the sale of these barrels can be modelled by the function $P(x)=-10 x^{2}+3500 x-66000$ where $P(x)$ is the profit in rupees and $x$ is the number of barrels made and sold.
i) When no barrels are produce what is a rofit or loss?
ii) What is the break even point?
(Zero profit point is called break even)


What is the profit /loss if
a) 175 barrels produce
b) 400 barrels produced
iii) What is maximum profit which can manufacturer earn?
37. Vijay is trying to find the average height of a tower near his house. He is using the properties of similar triangles. The height of Vijay's house if 20 m when Vijay's house casts a shadow 10 m long on the ground. At the same time, the tower casts a shadow 50 m long on the ground and the house of Ajay casts 20 m shadow on the ground.

i) What is the height of the tower?
ii) What is the height of Ajay's house?
iii) What will be the length of the shadow of the tower when Vijay's house casts a shadow of 12 m ?

## OR

When the tower casts a shadow of 40 m , same time what will be the length of the shadow of Ajay's house?
38. The Circus Arts Program is one of the most popular activities at Camp Lohikan. It brings a level of excitement and enthusiasm to the camp experience that can't be found at home or in school. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground such that the angle made by the rope with the ground level is $30^{\circ}$.

i) Find the height of the pole in the above situation.
ii) In the above situation, if $\mathrm{AB}=5 \mathrm{~m}$ and $\angle \mathrm{ACB}=30^{\circ}$ then find the length of the side $B C$.

## OR

In the above situation, if the height of pole is 3 m and the length of rope is 6 m then find $\angle A C B$
iii) If $\sqrt{3} \tan 2 \theta-3=0$ then $\theta=$ ?

